Neglecting higher power & equate the coe. of x.

$b_1 = 1$

$b_2 = a_1 = a_1 b_1$

$b_3 = a_1^2 + a_2 = a_1 b_2 + a_2 b_1$

$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1$

We have $f(x) = xe^x - 1 = 0$

$$f(x) = x\left[1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\right] - 1$$

$$f(x) = \left[x + x^2 + \frac{x^3}{\lfloor 2} + \frac{x^4}{\lfloor 3} + \cdots\right] - 1$$

Then by Ramanjan method we get

$$\left[1 - \left(x + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 3} + \cdots\right)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + \cdots$$

Now comparing this eq^n with roots of a_1, a_2, a_3 we get

$$\left[1 - (a_1 x + a_2 x^2 + a_3 x^3 + \cdots)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + \cdots$$

$a_1 = 1$, $a_2 = 1$, $a_3 = \frac{1}{\lfloor 2}$, $a_4 = \frac{1}{\lfloor 3}$, $a_5 = \frac{1}{\lfloor 4}$

also $b_1 = 1$

$b_2 = a_1 b_1 = 1 \cdot 1 = 1$

$b_3 = a_1 b_2 + a_2 b_1 = 1 \cdot 1 + 1 \cdot 1 = 2$

$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1 = 1 \cdot 2 + 1 \cdot 1 + \frac{1}{2} \times 1$

$b_4 = 7/2 = 3.5$

$b_5 = a_1 \cdot b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1$

$= 1 \times 3.5 + 1 \times 2 + \frac{1}{\lfloor 2} \times 1 + \frac{1}{6} \times 1$

$b_5 = 3.5 + 2 + 0.5 + 0.16$

$b_5 = 6.16$

$b_6 = a_1 b_5 + a_2 b_4 + a_3 b_3 + a_4 b_2 + a_5 b_1$

$= 1 \times 6.16 + 1 \times 3.5 + \frac{1}{\lfloor 2} \times 2 + \frac{1}{6} \times 1 + \frac{1}{24} \times 1$

$b_6 = 6.16 + 3.5 + 1 + 0.16 + 0.041$

$b_6 = 10.861$

Therefore $b_1 / b_2 = 1/1 = 1$

$b_2 / b_3 = 1/2 = 0.5$

$b_3 / b_4 = 2/3.5 = 0.571$

$b_4 / b_5 = 3.5/6.16 = 0.568$

$b_5 / b_6 = 6.16/10.861 = 0.5691$

Ques. find the roots of eq^n $\sin x = 1 - x$

$f(x) = 1 - x - \sin x$

$$= 1 - x - \left(x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 3} + \cdots\right)$$

$$f(x) = 1 - 2x + \frac{x^3}{\lfloor 3} - \frac{x^5}{\lfloor 5} + \frac{x^7}{\lfloor 7} + \cdots$$

Then by Ramangjan Method we get-

$$\left[1 - \left(2x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 3} + \cdots\right)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + \cdots$$

also $[1-(a_1x+a_2x^2+a_3x^3+a_4x^4+\ldots)]^{-1}$
$= b_1+b_2x+b_3x^2+\ldots$

Then $a_1=2$, $a_2=0$, $a_3=-\frac{1}{6}$, $a_4=0$

$a_5=\frac{1}{120}$, $a_6=0$, $a_7=\frac{1}{\lfloor 9}=\frac{1}{36288}$

$b_1=1$
$b_2=a_1=a_1b_1=2\cdot1=2$
$b_3=a_1b_2+a_2b_1=2\cdot2=4$
$b_4=a_1b_3+a_2b_2+a_3b_1$

$=2\times4+0+\left(-\frac{1}{6}\times1\right)$

$b_4=8-\frac{1}{6}=\frac{47}{6}=7.8$

$b_5=a_1b_4+a_2b_3+a_3b_2+a_4b_1$
$=2\times7.8-\frac{1}{6}\times4$
$=15.6-0.666$

$b_5=14.934$
$b_6=a_1b_5+a_2b_4+a_3b_3+a_4b_2+a_5b_1$

$b_6=2\times14.934+0-\frac{1}{6}\times4+0+\frac{1}{120}\times1$

$b_6=29.210$

There fore $b_1/b_2=1/2=0.5$
$b_2/b_3=2/4=0.5$
$b_3/b_4=4/1.8=0.51$

$b_4/b_5=\frac{7.8}{14.934}=0.52$

$b_5/b_6=\frac{14.934}{29.210}=0.511$

Sol^n of Non-linear eq^n

The sol^n of Non-linear eq^n is calculated by two methods known as
1. Itiration method
2. Newton's Raphson method

① Itiration Method → This method is used to solve the eq^n
$f(x,y)=0-(1)$ & $g(x,y)=0-(2)$

for this the func^n can be represent in the form of two coordinates x & y such that

$x=F(x,y)$ & $y=G(x,y)$

where the func^n F & G satisfy the cond^n

$\left|\frac{\partial F}{\partial x}\right|+\left|\frac{\partial F}{\partial y}\right|<1 \qquad \left|\frac{\partial G}{\partial x}\right|+\left|\frac{\partial G}{\partial y}\right|<1$

These are the sufficient cond^n of itteration method. The different coordinates can be calculated in the following manner.

$x_1=F(x_0,y_0)$, $y_1=G(x_0,y_0)$
$x_2=F(x_1,y_1)$, $y_2=G(x_1,y_1)$
$x_3=F(x_2,y_2)$, $y_3=G(x_2,y_2)$

$$x_n = F(x_{n-1}, y_{n-1}), \quad y_n = G(x_{n-1}, y_{n-1})$$

Que. Find the Solⁿ of eqⁿ $x = 0.2x^2 + 0.8$
$y = 0.3xy^2 + 0.7 = 1$

Solⁿ we have $F(x, y) = 0.2x^2 + 0.8$
$\Delta G(x, y) = 0.3xy^2 + 0.7$

Now on diff. w.r.t. 'x' & 'y' we get –

$$\frac{\partial F}{\partial x} = 0.4x \quad \& \quad \frac{\partial F}{\partial y} = 0$$

$$\frac{\partial G}{\partial x} = 0.3y^2, \quad \frac{\partial G}{\partial y} = 0.6xy$$

Now the value of x & y for non-linear eqⁿ then we choose $x = y = 1$
Then the component of x_0, y_0 will be $1/2$
then the condⁿ of iteration method will be

$$= \left|\frac{\partial F}{\partial x}\right| + \left|\frac{\partial F}{\partial y}\right| = 0.2 < 1 \quad \frac{0.4}{2} + 0 = 0$$

$$= \left|\frac{\partial G}{\partial x}\right| + \left|\frac{\partial G}{\partial y}\right| = 0.225 < 1$$

Hence the component of non-algebric eqⁿ will

$$x_1 = F(x_0, y_0) = 0.2 \times \frac{1}{4} + 0.8 = 0.85$$
$$y_1 = F(x_0, y_0) = 0.3 \times \frac{1}{8} + 0.7 = 0.74$$
$$x_2 = F(x_1, y_1) = 0.2 \times (.85)^2 + 0.8 = 0.94$$
$$y_2 = G(x_1, y_1) = 0.3 \times 0.85 \times (0.74)^2 + 0.7$$
$x_3 = F(x_2, y_2)$ $$= 0.8$$

Newton's Raphson Method : let x_0, y_0 be the approximation roots of the eqⁿ $f(x, y) = 0$ $g(x, y) = 0$ — ① If $x_0 + h$ & $y_0 + k$ is the root of the system then above eqⁿ can be written as

$$\left.\begin{aligned} f(x_0 + h), y + k) &= 0 \\ g(x_0 + h, y_0 + k) &= 0 \end{aligned}\right] — ①$$

Now by applying Taylor theorem in these funcⁿ we get

$$f_0 + h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0} + \cdots = 0 \quad \Big[\because x = x_0,\ y = y_0$$

IIIly. $$g_0 + h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0} + \cdots = 0$$

On neglecting higher order we get

$$-f_0 = h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0}$$

$$-g_0 = h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0}$$

Then the Jacobian of above eqⁿ can be written

$$J(f, g) = \begin{bmatrix} \partial f/\partial x & \partial f/\partial y \\ \partial g/\partial x & \partial g/\partial y \end{bmatrix} \neq 0$$

the component h & k in the form Jacobian can be written as

$$h = \frac{1}{J}\begin{bmatrix} -f & \partial f/\partial y \\ -g & \partial g/\partial y \end{bmatrix}$$

$$k = \frac{1}{J}\begin{bmatrix} \partial f/\partial x & -f \\ \partial g/\partial x & -g \end{bmatrix}$$

Then the new approximation is given as

$x_1 = x_0 + h$

$y_1 = y_0 + k$

Ques- Find the real root of the eq^n $x^2 - y^2 = 3$ —(1) & $x^2 + y^2 = 13$ —(2)

Sol^n $x^2 + y^2 = 13$

$2x_0^2 = 13$

$x_0 = \sqrt{6.5}$

The component of the approximation is calculated by replacing $x = y$ Then

$y_0 = x_0 = \sqrt{6.5} = 2.54$

Then $f(x_0) = (6.5)^2 - (6.5)^2 - 3 = -3$

$g(x_0) = (6.5)^2 + (6.5)^2 - 13 = 0 = -11.5$

Then the Jacobian can be calculated by diff. eq^n (1) & (2) w.r.t x & y we get

$\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = -2y$

$\frac{\partial g}{\partial x} = 2x$, $\frac{\partial g}{\partial y} = 2y$

Now
$$J[f,g] = \begin{vmatrix} 2\times 2.54 & -2\times 2.54 \\ 2\times 2.54 & 2\times 2.54 \end{vmatrix}$$

$$= \begin{vmatrix} 5.08 & -5.08 \\ 5.08 & 5.08 \end{vmatrix}$$

$= 2(5.08)^2 \neq 0$

The we know that

$-f_0(x_0) = h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0}$ —— (3)

Ily $-g(x_0) = h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0}$ —— (4)

Now replacing the value of $f(x_0)$, $g(x_0)$ $\partial f/\partial x_0$ & $\partial f/\partial y_0$ & $\partial g/\partial x_0$, $\frac{\partial g}{\partial y_0}$ we get

$\rightarrow$ $3 = h(5.08) - k\cancel{(5.08)}$

$0 = h(5.08) + k\cancel{(5.08)}$

$2h(5.08) = 3 \Rightarrow h = \frac{3}{10.16} = 0.29417$

$k = -0.29417$

Hence the Ist approximation can be written as

$x_1 = (x_0 + h)$

$y_1 = y_0 + k$

$x_1 = 2.54 + 0.294$